The role of the reservoir in the seismic analysis of concrete dams

M. Fanelli, Consultant, Milan, Italy
A. Fanelli, Informatics Expert, Milan, Italy
C. Gallimberti, Researcher, ENEL S.p.A./DSR/CRIS, Milan, Italy
P. Falumbo, Structural Engineer, ISMES S.p.A., Bergamo, Italy

It is well known that one of the energy dissipation mechanisms affecting the dynamic behaviour of a dam is associated with acoustic radiation from the reservoir bottom. The contribution of this mechanism to the overall damping coefficient is particularly important in the case of a full reservoir, and decreases markedly with lowering of the water level. The authors have developed a simple numerical procedure for evaluating this effect from a few basic data.

The key to the authors’ approach is the computation of a suitable universal, non-dimensional damping matrix, derived from idealised irrotational flow fields in the reservoir for “unit” motions of each node of the upstream dam face. With simple manipulations involving the physical constants of the bottom layers and the eigenfrequencies/eigenshapes of the dam, this damping matrix allows for reconstruction of the in-quadrature components of pressures acting on the upstream dam face during vibration.

It is thus possible to evaluate the energy dissipation during a cycle, and from this, the damping coefficient. In principle, iterations are required if full coupling is considered, because the eigenmodes become complex and eigen frequencies/eigenshapes are modified by the damping. In practice, the eigenvalues and eigenvectors computed by disregarding these slight damping-induced alterations constitute an excellent approximation, so that the fully coupled iterative procedure can be dispensed with.

The proposed method has been implemented in a software package for arch dam analysis (DESARC), and numerical tests have been carried out.

Satisfactory results have been obtained, as regards both the order of magnitude of the damping effect associated with the reservoir and the variation of it with the water level. Comparison with field test data, where available (and discounting other disturbing effects such as joint openings), are quite encouraging.

Some of the results are presented here to illustrate the potential of the proposed method.

The eigenfrequencies and eigenshapes of the vibration modes of a concrete dam are generally computed without taking into account the influence of the damping mechanisms that are always present in the physical system. The damping factor itself is usually not computed at all. It is determined either by comparison with similar structures (in similar local conditions) or by in situ tests. However, these can only be carried out after completion of the dam.

Comparisons between computed and experimental values for eigenfrequencies and eigenshapes of existing dams have shown that these dynamic characteristics are not perceptibly affected by the damping. However, knowledge of the damping factor itself can be important when carrying out, for example, a seismic analysis of a dam. If the dam is already built, the best way to acquire this knowledge is by in situ tests. If dynamic excitation tests are repeated on the same dam with different reservoir levels, it will be seen that the damping factor is not constant, but increases, if all other things are equal, with the increasing water level in the reservoir. This effect can be masked by seasonal influences, which can affect the opening or closing of construction joints near the surface, changing both the geometry and stiffness of the dam (and directly affecting the damping).

This paper describes a simple numerical procedure by which it is possible to estimate the damping factor associated with the presence of water in the reservoir, assuming the dam is monolithic (with the joints being completely closed).

It should not be forgotten that, in reality:

- The assumption of a monolithic dam may be unrealistically (especially in winter and when the water level is low).
- Other causes of damping are present (structural damping, radiation through the dam-foundation contact) which are not easily amenable to computation and which are beyond the scope of this paper. The relevant damping factor should be estimated separately and added to the reservoir-induced damping.

Let us now examine in detail the physical mechanism through which the presence of water in the reservoir induces additional damping.

The contact surface between the dam and the water (the upstream face or part of it) moves during a vibration cycle, exciting the impounded mass of water. A field of alternating motion and pressures is thus generated in the liquid. At the reservoir bottom, the oscillatory water pressures will excite the underlying layers of rock, creating acoustic waves which will propagate downwards, without appreciable reflection (if the rock mass is sufficiently homogeneous, as we will assume for simplicity). This “bottom radiation” effect will carry away energy from the vibrating system (dam/impounded water), causing a steady decrease of the vibration amplitudes. This decrease can be quantitatively expressed by a damping factor.

To be more precise, the coupling between the dam vibration and the acoustic waves in the bottom rock formations (coupling provided by the impounded mass of water) causes the eigenmodes to become “complex” (see below). This means that the vibrations will no longer be all in-phase (as in the idealized case of no damping, or when the damping matrix is a linear combination of the stiffness and mass matrix); likewise, the
hydrodynamic pressures on the wet portion of the upstream face will no longer be in-phase with the displacements, but will acquire small in-quadrature components. These in-quadrature components, being in opposition to the vibration velocities of the structure, will act in a negative way during a vibration cycle, causing a decrease in the potential/kinetic energy stored in the vibrating structure (damping).

In the next section a simplified mathematical model will be formulated of the energy transfer mechanism. The model will then provide an approximate evaluation of the associated damping factor.

It should be noted that the computational procedure allowing translation of the model into numerical software has been developed specifically for arch dams, within the framework of the DESARC package (for the preliminary analysis and design of arch dams) implemented by the authors and marketed by ISMES SpA [1 to 5]. However, the underlying concepts and formulations are quite general, and can easily be adapted to other types of concrete dams, within the limitations implied by the authors’ basic assumptions.

Preliminary numerical results show that the “bottom-radiation” damping factors computed using the proposed model are in approximate agreement with the estimates that can be obtained, although with a rather large margin of uncertainty, from actual in-situ dynamic tests performed with different water levels.\(^1\)

An interesting feature of these results is the very strong variation of the “bottom radiation” damping factor with the water level in the reservoir. In other words, the effect is quite appreciable for high levels of impoundment, but it decreases steeply with the lowering of the water level, so that the influence of this kind of damping on, say, the results of a seismic analysis will be appreciable only for the conditions of full, or nearly full, reservoir.

2. Approximate mathematical formulation

2.1 Hydrodynamic coupling

It will be assumed, for simplicity, in the following, that the central cantilever of an arch dam, of height \( H \) coincides with the depth of the reservoir (the case of partial impoundment can be dealt with by making straightforward modifications). The cantilever upstream facing is assumed, for greater clarity, to be vertical. The water flow field will be approximated with reference to a plane model, in the vertical plane through the axis of the central cantilever (see Fig. 1).

The liquid is characterized by its mass density \( \rho_L \) and its bulk compressibility \( c_L \); the bottom rock has mass density \( \rho_R \) and Young’s modulus \( E_R \).

For a generic motion of the upstream facing, only the horizontal components of displacements, denoted by the symbol \( \delta(x,z) \), will cause any fluid motion in the reservoir, under the assumption of inviscid liquid.

The deformed shape of the upstream face can be approximately expressed, for arbitrary values of \( z \), as a linear combination of \((n+1)\) nodal displacement \( \delta_j(0) \) \((1 \leq j \leq n+1)\), measured at \((n+1)\) equally spaced nodes (the vertical step being \( \Delta z = H/n \)), through the use of nodal shape functions \( N_j(z) \), see Fig.1:

\[
\delta(z,t) \equiv \sum_{j=1}^{n+1} \delta_j(t) N_j(z) \tag{2.1.1}
\]

where \( N_j(z) = 1; N_j(z) = 0 \) for \( j \neq k \); and,

\[
\sum_{j=1}^{n+1} N_j(z) \equiv 1.
\]

Now let each \( \delta_j(t) \) be a damped harmonic function of time \( t \):

\[
\delta_j(t) = \Delta_j e^{\sigma t} \tag{2.1.2}
\]

where \( \Delta_j \) is a complex amplitude, and:

\[
\sigma = i\omega - \lambda \quad (\lambda = \text{damping factor}) \tag{2.1.3}
\]

Then by using Eq. (2.1.1) for any arbitrary value of \( z \) \((0 \leq z \leq H)\) one gets:

\[
\delta(z,t) \equiv \sum_{j=1}^{n+1} \Delta_j N_j(z)e^{\sigma t} \tag{2.1.4}
\]

One should now define a flow field, inside the impounded liquid, that matches this motion of the upstream face. The simplest way to do this is to look for an irrotational motion characterized by a velocity potential \( \Phi(x,z,t) \):

\[
\Phi(x,z,t) \equiv \sum_{j=1}^{n+1} \delta_j \Phi_j(x,z) \tag{2.1.5}
\]

where, by using Eq.(2.1.2):

\[
\delta_j = \sigma \Delta_j e^{\sigma t} \tag{2.1.6}
\]

Here, it is convenient to work on non-dimensional unit solutions \( \phi_j(\xi,\zeta) \) (see Fig.1):

\[
\Phi_j(x,z) = H \phi_j(\xi,\zeta) \tag{2.1.7}
\]

Then it can be shown that Eq. (2.1.5) will satisfy the compatibility fluid/structure motion if the following boundary condition is imposed:

\[
\frac{\partial \phi_j}{\partial \zeta} = -N_j(\zeta) \quad \text{for} \quad \zeta = 0
\]

Two more boundary conditions are to be imposed on the free surface \((\xi=0)\) and at the reservoir bottom \((\xi=1)\), to ensure a constant pressure on the free surface and the radiation condition at the bottom:

\[
\phi_j(\xi,\zeta) = 0 \quad \text{for} \quad \zeta = 0
\]

\* Of course, for a correct comparison, the experimental damping factor for an empty reservoir should be subtracted from the overall damping factor measured with a full, or partially full, reservoir.
\[
\frac{\partial \phi_j}{\partial \zeta} = -H \frac{P_a}{\rho_c c_r} \frac{\partial \left( \phi e^{ij} \right)}{\partial t} = -\frac{P_a \sigma H}{\rho_c c_r} \phi_j \quad \text{for} \; \zeta = 1
\]
where:
\[
c_r = \left( \frac{E_r}{\rho_r} \right)^{1/2}
\]
which is the celerity of P-waves in the bed rock disregarding the Poisson effect.

The last boundary conditions, shown in Eq. (2.1.9), can in fact be derived from the bottom radiation condition with the assumption that acoustic waves in the bottom rock are quasi-plane P-waves (with horizontal wavefronts) radiating away to infinity in the vertical down-ward direction. The bottom pressure generated in the liquid by each of the unit solutions is (for unit velocity at node \( j \), and neglecting the kinetic term under the assumption of small displacements):
\[
-p_a \frac{\partial \phi_j}{\partial t} = H
\]
This value must in fact be equal to the acoustic pressure generated in the bottom rock when the vertical velocity of the bottom surface is \( \partial \phi_j / \partial \zeta \) (for unit velocity at node \( j \)), that is, equal to \( \rho_c c_r \left( \partial \phi_j / \partial \zeta \right) \). This leads to the second part of Eq. (2.1.9).

The indefinite partial differential equation valid for each of the \( \phi_j \) inside the fluid domain can be written as:
\[
\nabla^2 \phi_j = \frac{\partial^2 \phi_j}{\partial \zeta^2} + \frac{\partial^2 \phi_j}{\partial \xi^2} = \left( \frac{\sigma H}{c_a} \right)^2 \phi_j
\]
where:
\[
c_a = \left( \frac{E}{\rho_a} \right)^{1/2}
\]
which is equal to sound celerity in water \( \equiv 1440 \) m/s.

In the following we will accept the rather artificial assumption \( c_a = \infty \), that is:
\[
\nabla^2 \phi_j \equiv 0
\]
Even if this is in principle inconsistent with attributing a finite celerity \( c_r \) to the P-waves in the bottom rock, this is justified (on purely empirical grounds) by the fact that the “virtual mass” effect computed on the basis of the “unit solutions” \( \phi_j \) satisfying Eq. (2.1.13) (see further on) gives variations of eigenfrequencies with the water level in excellent agreement with experimental values [Fanelli and Fanelli, 1992]. (Also, eigenshapes computed in this way are in good agreement with in-situ test results; their variations with the water level cannot be appreciated, however, with the same degree of accuracy and sensitivity that one can achieve for eigenfrequencies).

As a final step in our formulation of the hydrodynamics of the impounded liquid during vibration, it should be stressed that each \( \phi_j \) is a complex function of \( \xi, \zeta \):
\[
\phi_j = p_j - iq_j \quad (i = \sqrt{-1})
\]
where \( p_j(\xi, \zeta) \) and \( q_j(\xi, \zeta) \) are now real variables.

By proposing further:
\[
c = \frac{c_a \sigma_r}{\rho_a} = \frac{\sqrt{E_r \rho_r}}{\rho_a} \quad (2.1.15)
\]
the second part of Eq. (2.1.9) becomes, see Eq. (2.1.3):
\[
\frac{\partial p_j}{\partial \zeta} - i \frac{\partial q_j}{\partial \zeta} = -i \frac{\lambda H}{c} \left( p_j - iq_j \right) H
\]
and, by separating the real from the imaginary parts:
\[
\frac{\partial p_j}{\partial \zeta} = -\frac{\omega H}{c} q_j + \frac{\lambda H}{c} p_j
\]
\[
\frac{\partial q_j}{\partial \zeta} = \frac{\omega H}{c} p_j + \frac{\lambda H}{c} q_j
\]
\[
(2.1.16)
\]
for \( \zeta = 1 \) with \( \nabla^2 p_j = \nabla^2 q_j = 0 \) and \( p_j(\xi, 0) = q_j(\xi, 0) = 0 \); with respect to Eq. (2.1.8) for \( p_a \), and \( \partial q_j / \partial \zeta = 0 \), for \( \zeta = 0 \). If, as it usually happens, the non-dimensional parameters \( \omega H / c, \lambda H / c \) are of such an order of magnitude that:
\[
\frac{\lambda H}{c} \ll \frac{\omega H}{c} \quad (2.1.17)
\]
then as a first approximation one may take the following as the boundary condition for \( \zeta = 1 \) (radiation condition):
\[
\frac{\partial p_j}{\partial \zeta} \equiv -\frac{\omega H}{c} q_j \equiv 0
\]
\[
\frac{\partial q_j}{\partial \zeta} \equiv \frac{\omega H}{c} p_j
\]
\[
(2.1.18)
\]
Now the problem of determining the “unit solution” can easily be solved numerically (for example, by a straightforward finite element computation); moreover, if one does not directly solve \( p_j \) and \( q_j \), but rather \( p_j \) and \( q_j^* \), with the boundary condition for \( \zeta = 1 \):
\[
\begin{aligned}
\frac{\partial p_j}{\partial \zeta} &= 0 \\
\frac{\partial q_j}{\partial \zeta} &= p_j \\
\frac{\partial q_j^*}{\partial \zeta} &= q_j^*
\end{aligned}
\]
\[
(2.1.19)
\]
by stating:
\[
q_j^* = \frac{\omega H}{c} q_j
\]
and if \( n \) is kept constant (for example, \( n = 10 \), it is seen that the \( n + 1 \) couples of “unit solutions” \( p_j(\xi, \zeta), q_j^*(\xi, \zeta) \) can be built up numerically once and for all, independently of the parameter \( \omega H / c \).

In this sense \( q_j^* (\xi, \zeta) = q_j^0 \) defines a “universal” damping matrix, as described in the next section.

Figs. 2 and 3 show some of the numerical results for \( p_j \) and \( q_j^0 \).

\[\text{† Eq. (2.1.17) defines a partial differential coupled problem to be solved for } p_j(\xi, \zeta), q_j^0(\xi, \zeta).\]
\[\text{‡ Typically } \omega H / c \text{ is of the order of } 0.2, \lambda H / c \text{ is of the order of } 0.004.\]
Note that the matrix \([p_{0i}]\) represents, apart from the factor \(1/\rho_d H\Delta z\) (see footnote), the "virtual mass" (or "added mass") matrix taken into account the fluid-structure interaction in the case of a rigid bottom. We will use it also in the present formulation with the assumption that the perturbation caused to eigenfrequencies and eigenshapes by damping associated with bottom compliance are quite small in relative terms; an assumption, as already mentioned, which is borne out by comparison of computed values with in-situ test results.

It should be pointed out that the virtual mass matrix thus defined is a "consistent" full matrix, in contrast with the "added masses" of the classical Westergaard formulation [1931], which basically assumes a uniform value of the \(\Delta z\) and thus leads to a diagonal, lumped, inconsistent mass matrix.

2.2 The structural damped-oscillation problem

For convenience, reference is made to a finite-element discretization model. In terms of the nodal displacement vector, \((\delta_j)\), the equation of motion can be written as:

\[
[K] \dot{\delta}_j + [M] \ddot{\delta}_j = \Delta z \left[ p_{j} \right] = -\rho_d \frac{\Delta z}{2} \left[ \frac{\partial \Phi_j}{\partial t} \right]
\] (2.2.1)

where \([K]\) = stiffness matrix, \([M]\) = mass matrix of the structure proper, \([p_{j}]\) = vector of the hydrodynamic pressures acting on the upstream face nodes. From Eqs. (2.1.5), (2.1.6), (2.1.7), (2.1.14) we obtain:

\[
[K] \dot{\delta}_j + \sigma^2 [M] \ddot{\delta}_j = -\rho_d \frac{H \Delta z}{2} \left[ p_{j} \right] = -\rho_d \frac{H \Delta z}{2} \left[ \frac{\partial \Phi_j}{\partial t} \right] \left[ \dot{\delta}_j \right]
\] (2.2.2)

or,

\[
[K] \dot{\delta}_j + \sigma^2 [M] \ddot{\delta}_j = i \rho_d \frac{H \Delta z}{2} \left[ \frac{\partial \Phi_j}{\partial t} \right] \left[ \dot{\delta}_j \right]
\] (2.2.3)

which is a homogeneous linear system in the complex domain, defining a complex eigenvalue/eigenvector problem, the complex eigenvalue unknown being, see Eq. (2.1.3), \(\sigma = \alpha \pm i \beta\), and the eigenvector (also a complex unknown) being \((\hat{\delta}_j)\).

† The factor 1/2 derives from the fact that \(\delta\) is not uniform along the arch; its average is about half the value at the central cantilever.

†† \([P_j] = [R_j]+[Q_j]\). \([R_j]\) being the real components of \([P_j]\) and \([Q_j]\) the imaginary ones.

††† It is evident that in general \([C] = p_d H \Delta z / 2 \alpha \left[ \dot{\delta}_j \right] \left[ \frac{\partial \Phi_j}{\partial t} \right] \begin{bmatrix} \delta_j \end{bmatrix} = C \left[ \ddot{\delta}_j \right] + M \left[ \dot{\delta}_j \right] = 0\) also, the role of the product:

\[
\rho_d \frac{H \Delta z}{2} \left[ p_{j} \right]
\]

as a "virtual" (or "added") mass matrix representing the inertial effect of the fluid/structure interaction is clearly recognized.

It should also be noted that, with the simplifying assumptions made so far, \([p_{0i}]\) and \([q_{0i}]\) are indeed "universal" matrices which can be computed once and for all. But, if need be, nothing prevents the designer from keeping the more general conditions [compressibility of water, and the coupled problem for \(p_{0i} (\xi, \zeta)\) and \(q_{0i} (\xi, \zeta)\)] and on this basis computing more accurate \([p_{0i}]\) and \([q_{0i}]\) matrices for the particular case.

Now Eq. (2.2.3) can be solved in an iterative way starting from a "first approximation" solution. For the latter, one assumes for \(\alpha\) and \((\Delta z)\) the values that can be computed in the absence of damping, that is, assuming \(E_i = \infty, c = c_i = \infty\) and consequently \(q_i = 0\), see Eq. (2.1.20); the right-hand side of Eq. (2.2.3) is a null vector in this case. With these assumptions, one can compute the in-quadrature components of the pressures acting on the upstream face:

\[
\left[ Q_i \right] \equiv -\rho_d H\omega \frac{\partial \Phi_j}{\partial t} \left[ \dot{\delta}_j \right] = -i \rho_d H\omega \frac{\partial \Phi_j}{\partial t} \left[ \dot{\delta}_j \right] = 0
\] (2.2.4)

where \(c\) is now given by Eq. (2.1.15) with the proper value of \(E_i\); then the work done during a vibration cycle by these pressures on the cantilever is:

\[
L_r \equiv \frac{2}{\rho_d} \int_{0}^{\infty} \left[ Q_i \right]^{T} \frac{\partial \dot{\delta}_j}{\partial t} \, dt
\] (2.2.5)
With straightforward manipulations, Eq. (2.2.5) gives:

$$L_r = -\pi \alpha^2 \rho_a H \Delta \frac{\alpha H}{c} \left\{ \Delta_j \right\}^T \left\{ q_k^* \right\} \left\{ \Delta_j \right\} \frac{1}{2}$$

(2.2.6)

The work given by Eq. (2.2.5) must be equal to the variation undergone during the same cycle by the maximum kinetic energy $W_c$ of the vibrating system (concrete + fluid) of the central cantilever:

$$W_c \equiv \frac{\alpha}{2} \left\{ \Delta \right\}^T M \left\{ \Delta \right\}$$

(2.2.7)

where:

$$[M] \equiv [M] + \rho_a \frac{H \Delta}{2} [\rho]$$

(2.2.8)

see Eq. (2.2.3); then the variation of $W_c$ during a vibration period is:

$$dW_c \equiv \omega^2 \left\{ d\Delta \right\}^T [M] \left\{ \Delta \right\}$$

(2.2.9)

where:

$$\left\{ d\Delta \right\} \equiv -\frac{2\pi d}{\omega} \lambda \left\{ \Delta \right\}$$

(2.2.10)

see Eqs. (2.1.2) and (2.1.3).

From Eqs. (2.2.9) and (2.2.10) one obtains:

$$dW_c \equiv -2\pi \alpha \lambda \omega^2 \left\{ d\Delta \right\}^T [M] \left\{ \Delta \right\}$$

(2.2.11)

From $dW_c = 2L_r$, an estimate can be obtained for $\lambda$:

$$\lambda \equiv \frac{\omega}{2} \rho_a H \Delta \frac{\alpha}{c} \left\{ \Delta \right\}^T q_k^* \left\{ \Delta \right\}$$

(2.2.12)

With this first approximation, one can enter an iterative cycle for solving Eq. (2.2.3); but usually Eq. (2.2.12) is accurate enough for practical purposes, as are the "undamped" estimates for $\omega$ and $\Delta$.

From the expression of $\lambda$ in Eq. (2.2.12), and from $\lambda = \beta \omega$, $\beta$ being the ratio of damping factor $\lambda$ to the critical damping $\omega$, it follows for $\beta$:

$$\beta \equiv \frac{\omega}{2c} \rho_a H \Delta \frac{\left\{ \Delta \right\}^T q_k^* \left\{ \Delta \right\}}{\left\{ \Delta \right\}^T [M] \left\{ \Delta \right\}}$$

(2.2.13)

To verify this first approximation, the values should be in accordance with:

$$\lambda \ll \omega, \quad \beta \ll 1 \quad \text{and} \quad \frac{\omega H}{c} \ll 1$$

(2.2.14)

Fig. 4 shows some numerical results obtained for $\beta$ as a function of the impoundment level in the case of the well known Talvachia dam [1991*]. Some indications taken from the results of in-situ dynamic tests, carried out in different water conditions, are also shown in the figure. Taking into account the uncertainties affecting the experimental estimate of the bottom-compliance-induced damping, the agreement between theoretical and empirical results appears to be satisfactory.

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### 3. Influence of mud layers on the reservoir bottom

An additional circumstance that can affect the intensity of energy losses during free vibration is the presence of dissipating layers on the reservoir bottom, such as muddy deposits. Only one such layer will be considered here. Making simplifying assumptions similar to the ones already adopted in the preceding sections, it is possible to take into account this situation with the model. Additional assumptions (mainly adopted to achieve greater clarity of formulation) concern the thickness $s$, of the deposits, which is assumed to be small compared with $H$ (see Fig. 5):

$$s \ll H$$

(3.1)

and the type of energy dissipation within the muddy layer, represented by adopting a complex value for the velocity inside the layer, is:

$$c_1 = c_0(1 + i \varepsilon), \quad \varepsilon \ll 1$$

(3.2)

In the muddy layer, account should be taken of the existence of quasi-plane compressional waves travelling both upwards and downwards. In the rock, only downward travelling acoustic plane waves will be considered, as in the preceding case. The following values need to be defined for the muddy layer:

- $\rho_1$ = mass density of mud
- $s$ = thickness of layer
- $c_0$ = velocity in the mud
- $\varepsilon$ = damping in the mud as a fraction of critical damping, alternatively, the kinematic viscosity $\nu$ can be defined:

$$\varepsilon = \frac{2}{3} \nu$$

from these, the following parameters are computed:

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**Fig. 5. Reference schematic for the influence of reservoir sediment deposits.**
Continuity of velocities and equilibrium of pressures are imposed at both interfaces (water/mud and mud/rock). A complex velocity potential is assumed for the fluid flow, as before.

The mathematical developments are omitted for brevity; it is sufficient to outline the results. By first computing the following quantities:

\[ d = Sh^2 \left( \frac{\varepsilon_1 \omega x}{c_0} \right) \left( \sin^2 \left( \frac{\omega x}{c_0} \right) + \frac{\cos^2 \left( \frac{\omega x}{c_0} \right)}{k^2} \right) + \]
\[ Ch^2 \left( \frac{\varepsilon_1 \omega x}{c_0} \right) \left( \cos^2 \left( \frac{\omega x}{c_0} \right) + \frac{\sin^2 \left( \frac{\omega x}{c_0} \right)}{k^2} \right) + \]
\[ + \frac{1}{k} Sh \left( \frac{2\varepsilon_1 \omega x}{c_0} \right) \]

\[ a = \frac{1}{2d} \left( 1 + \frac{1}{k^2} Sh \left( \frac{2\varepsilon_1 \omega x}{c_0} \right) \right) + \]
\[ + \frac{1}{kd} \left( Ch^2 \left( \frac{\varepsilon_1 \omega x}{c_0} \right) + S h^2 \left( \frac{\varepsilon_1 \omega x}{c_0} \right) \right) \]

\[ b = \frac{1 - \frac{1}{k^2} \sin \left( \frac{2\omega x}{c_0} \right)}{2d} \]

\[ F^* = \frac{a oH}{(a^2 + b^2)c^*} \]

then the same algorithm as in the preceding case, Eq. (2.2.12), can be used, by simply putting \( F^* \) in the place of the parameter \( aoH \)/\( c \), or \((a^2 + b^2)/c^*\) in the place of \( c \).

It should be pointed out that, at the time of writing, the authors have no knowledge of the existence of experimental data that could be used to validate the simplified model described in this section.

**Conclusions**

The simplified mathematical model, and the algorithms derived, aim to obtain a numerical estimate of the damping of free vibrations associated with the dynamic coupling dam/reservoir/rock bottom. The results obtained so far using this approach have shown the following:

1. The order of magnitude of the damping factor obtained is in satisfactory agreement with the results of in-situ tests, within the quite large range of uncertainty affecting the latter.
2. On the basis of this approach, the reservoir-induced damping effect appears to be very strongly dependent on the water level in the reservoir; it is appreciable mainly for a full, or nearly full, reservoir, and decreases greatly with the lowering of the free surface.

Other causes of damping (structural damping in the dam material, radiation from the dam/foundation contact surface) are not considered in this study.

Also, the influence of seasonal conditions, which can affect both eigenfrequencies and damping, (for a given water level) through the opening or closing of construction joints, are not taken into account here.

Further research efforts could be aimed at including these effects, and at improving the quality of experimental databases to validate the preliminary results which have been described.

In seismic analysis, the global damping factor should of course be used. That means that, the reservoir-induced damping factor, determined by the present approach, should be added to an estimate of the damping factor caused by the other dissipative mechanisms of the physical system (structural damping in the dam material, friction in the joints, radiation at the dam/foundation interface and so on). These energy dissipation effects will also be dependent on the water level and the seasonal conditions, through the state of compression in the rock and the degree of opening of construction joints; it can be surmised, however, that this dependence will be less strong than for the reservoir-induced damping.

It is evident that further experimental and theoretical research would be very useful with a view to increasing knowledge of these relationships, not only for scientific purposes, but to enable more reliable assessments to be made of the seismic safety of dams.

**References**


Michele Fanelli received his free-teaching professorship in Construction Techniques in 1970. Up until his retirement in 1993, he was the Director of the ENEL Centre for Hydraulic and Structural Research. Prof. Fanelli is now an independent consultant.

Via L. B. Alberti 5, 20149 Milan, Italy.

Alberto Fanelli is a Co-Director of ICE (Internet Centre of Excellence) S.r.l. in Milan. Previously he worked with ISMES S.p.A. on the development of the DESARC code using neural network optimization.

ICE S.r.l., Via Montecatini 13, 20144 Milan, Italy.

Carlo Gallimberti has been involved with the monitoring of the structural behaviour of dams since 1978. At present he works as an expert on measurement instrumentation at the ENEL Centre for Hydraulic and Structural Research.

ENEL/DSR/CRS, Via L. Omato 90/14, 20162 Milan, Italy.

Pasquale Palumbo is Senior Engineer in the Mathematical Modelling Department of ISMES SpA. He has worked with ISMES in the field of dynamic analysis since 1987. His special area of interest is fluid-structure and soil-structure interaction.

ISMES S.p.A., Viale G. Cesare 29, 24100 Bergamo, Italy.